DAY TWENTY THREE

Unit Test 3

(Trigonometry)

1 The range of $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$, is

(a) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (b) $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

(c) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (d) None of these

2 From a point a m above a lake the angle of elevation of a cloud is α and the angle of depression of its reflection is β. The height of the cloud is

(a) $\frac{a \sin(\alpha + \beta)}{\sin(\alpha - \beta)}$ m (b) $\frac{a \sin(\alpha + \beta)}{\sin(\beta - \alpha)}$ m

(c) $\frac{a \sin(\beta - \alpha)}{\sin(\alpha + \beta)}$ m (d) None of these

3 If $\sum_{i=1}^{2n} \cos^{-1} x_i = 0$, then $\sum_{i=1}^{2n} x_i$ is equal to

(a) n (b) $2n$

(c) $\frac{n(n+1)}{2}$ (d) None of these

4 The value of x for which $\cos^{-1}(\cos 4) > 3x^2 - 4x$, is

(a) $\left(0, \frac{2 + \sqrt{6\pi - 8}}{3}\right)$

(b) $\left(\frac{2 - \sqrt{6\pi - 8}}{3}, 0\right)$

(c) $(-2, 2)$

(d) $\left(\frac{2 - \sqrt{6\pi - 8}}{3}, \frac{2 + \sqrt{6\pi - 8}}{3}\right)$

5 If $\alpha \le \sin^{-1} x + \cos^{-1} x + \tan^{-1} x \le \beta$, then

(a) $\alpha = 0$ (b) $\beta = \frac{\pi}{2}$ (c) $\alpha = \frac{\pi}{4}$ (d) $\beta = \frac{\pi}{3}$

6 The angles of elevation of the top of a tower at the top and the foot of a pole of height 10 m are 30° and 60°, respectively. The height of the tower is

(a) 10 m (b) 15 m

(c) 20 m (d) None of these

7 The maximum value of $12 \sin\theta - 9 \sin^2 \theta$ is

(a) 3 (b) 4

(c) 5 (d) None of these

8 The minimum value of the expression $\sin\alpha + \sin\beta + \sin\gamma$, when α, β and γ are real numbers satisfying $\alpha + \beta + \gamma = \pi$, is

(a) -3 (b) negative (c) positive (d) zero

9 If $\cos^{-1} \sqrt{x} + \cos^{-1} \sqrt{1 - x} + \cos^{-1} \sqrt{1 - y} = \pi$, then the value of y is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 1 (d) $\frac{1}{4}$

10 If α and β are the roots of the equation $5 \cos\theta + 4 \sin\theta = 3$, then $\cos(\alpha + \beta)$ is equal to

(a) $\frac{9}{40}$ (b) $\frac{9}{41}$ (c) $\frac{3}{10}$ (d) $\frac{21}{31}$

11 The angular depressions of the top and the foot of a chimney as seen from the top of a second chimney, which is 150 m high and standing on the same level as the first are θ and ϕ, respectively. If $\tan\theta = \frac{4}{3}$ and $\tan\phi = \frac{5}{2}$, then the distance between their tops is

(a) 120 m (b) 110 m

(c) 100 m (d) None of these

12 The elevation of the hill from a place P due West of it is 60° and at a place Q due South of it is 30°. If the distance PQ be 200 m, then the height of the hill is

(a) 109.54 m (b) 108.70 m

(c) 110.6 m (d) None of these

13 $\cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(\cos\frac{7\pi}{5} - \sin\frac{2\pi}{5}\right)\right\}$ is equal to

(a) $\frac{13\pi}{20}$ (b) $\frac{21\pi}{20}$

(c) $\frac{33\pi}{20}$ (d) None of these

14 $\sec^2\theta = \dfrac{4ab}{(a+b)^2}$, where $a, b \in R$ is true if and only if

(a) $a+b \neq 0$ (b) $a=b, a\neq 0$
(c) $a=b$ (d) $a\neq 0, b\neq 0$

15 If $x = \cos^{-1}(\cos 4)$ and $y = \sin^{-1}(\sin 3)$, then which of the following conditions holds?

(a) $x - y = 1$ (b) $x + y + 1 = 0$
(c) $x + 2y = 2$ (d) $\tan(x+y) = -\tan 7$

16 If $\log_2 x \geq 0$, then $\log_{1/\pi}\left\{\sin^{-1}\dfrac{2x}{1+x^2} + 2\tan^{-1}x\right\}$ is equal to

(a) $\log_{1/\pi}(4\tan^{-1}x)$ (b) 0
(c) -1 (d) None of these

17 If A lies in the third quadrant and $3\tan A - 4 = 0$, then $5\sin 2A + 3\sin A + 4\cos A$ is equal to

(a) 0 (b) $-\dfrac{24}{5}$
(c) $\dfrac{24}{5}$ (d) $\dfrac{48}{5}$

18 The minimum value of $27^{\cos x} + 81^{\sin x}$ is

(a) $\dfrac{2}{3\sqrt{3}}$ (b) $\dfrac{1}{3\sqrt{3}}$
(c) $\dfrac{2}{9\sqrt{3}}$ (d) None of these

19 If $\sin^2 x + a\sin x + 1 = 0$ has no real number solution, then

(a) $|a| \geq 2$ (b) $|a| \geq 1$
(c) $|a| < 2$ (d) None of these

20 If $\sin\theta = n\sin(\theta + 2\alpha)$, then $\tan(\theta+\alpha)$ is equal to

(a) $\dfrac{n+1}{n-1}\tan\alpha$ (b) $\dfrac{1+n}{1-n}\tan\alpha$
(c) $\dfrac{n}{1+n}\tan\alpha$ (d) None of these

21 $\sin^6 x + \cos^6 x$ lies between

(a) $\dfrac{1}{4}$ and 1 (b) $\dfrac{1}{4}$ and 2
(c) 0 and 1 (d) None of these

22 If $n = \dfrac{\pi}{4\alpha}$, then $\tan\alpha \cdot \tan 2\alpha \cdot \tan 3\alpha \ldots \tan(2n-1)\alpha$ is equal to

(a) 1 (b) -1
(c) ∞ (d) None of these

23 The ratio of the greatest value of $2 - \cos x + \sin^2 x$ to its least value is

(a) $\dfrac{1}{4}$ (b) $\dfrac{9}{4}$ (c) $\dfrac{13}{4}$ (d) $\dfrac{17}{4}$

24 If $m = a\cos^3\theta + 3a\cos\theta\sin^2\theta, n = a\sin^3\theta + 3a\cos^2\theta\sin\theta$, then the value of $(m+n)^{2/3} + (m-n)^{2/3}$ is

(a) $2a^{2/3}$ (b) $a^{2/3}$ (c) $a^{3/2}$ (d) $4a^{2/3}$

25 If $a\sin^2\alpha - \dfrac{1}{a}\text{cosec}^2\alpha = 0, 0 < \alpha < \dfrac{\pi}{2}$, then $\cos^2\alpha + 5\sin\alpha\cos\alpha + 6\sin^2\alpha$ is equal to

(a) 5 (b) $\dfrac{a^2+5a+6}{a^2}$
(c) $\dfrac{a^2-5a+6}{a^2}$ (d) None of these

26 If $\tan\dfrac{x}{2} = \text{cosec}\,x - \sin x$, then $\tan^2\dfrac{x}{2}$ is equal to

(a) $2-\sqrt{5}$ (b) $\sqrt{5}-2$
(c) $(9-4\sqrt{5})(2+\sqrt{5})$ (d) $(9+4\sqrt{5})(2-\sqrt{5})$

27 If $0° < \theta < 180°$, then $\sqrt{2+\sqrt{2+\sqrt{2+\ldots+\sqrt{2(1+\cos\theta)}}}}$ (where, number of 2's is n) is equal to

(a) $2\cos\left(\dfrac{\theta}{2^n}\right)$ (b) $2\cos\left(\dfrac{\theta}{2^{n-1}}\right)$
(c) $2\cos\left(\dfrac{\theta}{2^{n+1}}\right)$ (d) None of these

28 In ΔABC, $\tan A + \tan B + \tan C = 6$, $\tan B\tan C = 2$, then $\sin^2 A : \sin^2 B : \sin^2 C$ is equal to

(a) $\dfrac{9}{10}:\dfrac{5}{10}:\dfrac{8}{10}$ (b) $\dfrac{9}{10}:\dfrac{7}{10}:\dfrac{8}{10}$
(c) $\dfrac{9}{10}:\dfrac{8}{10}:\dfrac{7}{10}$ (d) None of these

29 If $0 \leq x \leq 3\pi$, $0 \leq y \leq 3\pi$ and $\cos x \cdot \sin y = 1$, then the possible number of values of the ordered pair (x, y) is

(a) 6 (b) 12 (c) 8 (d) 15

30 The general solution of the equation $1 + \sin^4 2x = \cos^2 6x$ is

(a) $\dfrac{n\pi}{3}$ (b) $\dfrac{n\pi}{2}$ (c) $3n\pi$ (d) None of these

31 The equation $2\cos^2\dfrac{x}{2}\sin^2 x = x^2 + x^{-2}, 0 < x \leq \dfrac{\pi}{2}$ has

(a) no real solution
(b) a unique real solution
(c) finitely many real solutions
(d) infinitely many real solutions

32 The n poles standing at equal distance on a straight road subtend the same angle α at a point O on the road. If the height of the largest pole is h and the distance of the foot of the smallest pole from O is a, the distance between two consecutive poles is

(a) $\dfrac{h\cos\alpha - a\sin\alpha}{(n-1)\sin\alpha}$ (b) $\dfrac{h\cos\alpha + a\sin\alpha}{(n-1)\sin\alpha}$
(c) $\dfrac{h\cos\alpha - a\sin\alpha}{(n+1)\sin\alpha}$ (d) $\dfrac{a\cos\alpha - h\sin\alpha}{(n-1)\sin\alpha}$

33 The solution of the equation

$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\dfrac{8}{31}\right)$ is

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{4}$ or -8
(c) -8 (d) None of these

34 The solution of $\cos^{-1} x\sqrt{3} + \cos^{-1} x = \frac{\pi}{2}$, is

(a) $-\frac{1}{2}$ (b) $-\frac{1}{2}$ or $\frac{1}{2}$
(c) $\frac{1}{2}$ (d) None of these

35 The number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$, is

(a) 0 (b) 1 (c) 2 (d) 3

36 If $\cot^2 x + \text{cosec } x - a = 0$ has atleast one solution, then complete set of value of a is

(a) $[-1, \infty)$ (b) $(-1, 1)$ (c) $(-\infty, 1)$ (d) $(-\infty, 1]$

37 From the top of the cliff 400 m high the top of a tower was observed at an angle of depression 45° and from the foot of the tower the top of the cliff was observed at an angle of elevation 60°. The height of the tower is

(a) 169.06 m (b) 179.2 m
(c) 180.0 m (d) None of these

38 If $\cos 5\theta = P\cos\theta - 20\cos^3\theta + Q\cos^5\theta + R$, then $P + Q + R$ is equal to

(a) 21 (b) 18 (c) 15 (d) 31

39 In ΔABC, if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$, then the triangle is

(a) right angled (b) obtuse angled
(c) equilateral (d) isosceles

40 If the area of a ΔABC be λ, then $a^2 \sin 2B + b^2 \sin 2A$ is equal to

(a) 2λ (b) λ
(c) 4λ (d) None of these

41 If $\cos 2x + 2\cos x = 1$, then the value of $\sin^2 x(2 - \cos^2 x)$ is

(a) 0 (b) 1 (c) 2 (d) -1

42 If $\sin x(1 + \sin x) + \cos x(1 + \cos x) = A$ and $\sin x(1 - \sin x) + \cos x(1 - \cos x) = B$, then the value of $A^2 - 2A - \sin 2x = B^2 + 2B - \sin 2x$ is

(a) 1 (b) 2 (c) 3 (d) 0

43 If the sides a, b and c of a ΔABC are roots of the equation $x^3 - 15x^2 + 47x - 82 = 0$, then the value of $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$ is

(a) $\frac{225}{164}$ (b) $\frac{131}{164}$ (c) $\frac{131}{82}$ (d) $\frac{169}{82}$

44 If in a ΔABC, $a^4 + b^4 + c^4 - 2b^2c^2 - 2c^2a^2 = 0$. Then, the angle c is

(a) 30° or 150° (b) 45° or 135°
(c) 60° or 120° (d) None of these

45 Let $f(\theta, \alpha) = 2\sin^2\theta + 4\cos(\theta + \alpha)\sin\theta\sin\alpha + \cos 2(\theta + \alpha)$. Then, the value of $f\left(\frac{\pi}{3}, \frac{\pi}{4}\right)$ is

(a) 0 (b) 1 (c) 2 (d) 3

46 Which among the following is/are correct statement/statements?

I. The general value of θ satisfying the equations $\sin^2\theta = \sin^2\alpha$, $\cos^2\theta = \cos^2\alpha$ and $\tan^2\theta = \tan^2\alpha$ is given by $\theta = n\pi \pm \alpha$

II. The general value of θ satisfying equations $\sin\theta = \sin\alpha$ and $\cos\theta = \cos\alpha$ simultaneously is given by $\theta = 2n\pi \pm \alpha$, $n \in Z$.

(a) I is correct (b) II is correct
(c) Both I and II are correct (d) Both I and II are incorrect

Directions (Q. Nos. 47-50) *Each of these questions contains two statements : Statement I and Statement II. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.*

(a) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

47 **Statement I** If $\frac{1}{2} \le x \le 1$, then $\cos^{-1} x - \sin^{-1}\left[\frac{x}{2} + \frac{\sqrt{3 - 3x^2}}{2}\right]$ is equal to $-\frac{\pi}{3}$.

Statement II $\sin^{-1}(2x\sqrt{1 - x^2}) = 2\sin^{-1} x$, if $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$.

48 Suppose any angle is satisfied both equations, then it is said to be solution of an equation.

Statement I The number of solutions of the pair of equations

$$2\sin^2\theta - \cos 2\theta = 0$$
and
$$2\cos^2\theta - 3\sin\theta = 0$$
in the interval $[0, 2\pi]$ is two.

Statement II If $2\cos^2\theta - 3\sin\theta = 0$, then θ cannot lie in III or IV quadrant.

49 **Statement I** If $2\sin\left(\frac{\theta}{2}\right) = \sqrt{1 + \sin\theta} + \sqrt{1 - \sin\theta}$, then $\frac{\theta}{2}$ lies between $2n\pi + \frac{\pi}{4}$ and $2n\pi + \frac{3\pi}{4}$.

Statement II If $\frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$, then $\sin\frac{\theta}{2} > 0$.

50 Suppose a person is standing on a ground and see the tower, then they make an angle of elevation.

Statement I A tower subtends angles $\alpha, 2\alpha$ and 3α respectively at points A, B and C all lying on a horizontal line through the foot of AB tower, then $AB : BC = \sin 3\alpha : \sin\alpha$.

Statement II In ΔABC, if $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, then ΔABC is an equilateral triangle.

ANSWERS

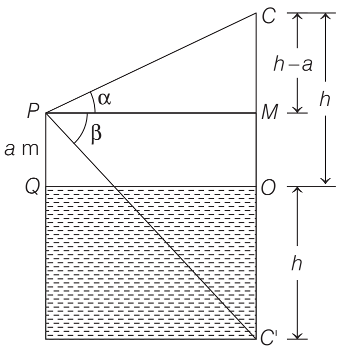
1 (c)	**2** (b)	**3** (b)	**4** (d)	**5** (a)	**6** (b)	**7** (b)	**8** (c)	**9** (c)	**10** (b)
11 (c)	**12** (a)	**13** (d)	**14** (b)	**15** (d)	**16** (c)	**17** (a)	**18** (c)	**19** (c)	**20** (b)
21 (a)	**22** (a)	**23** (c)	**24** (a)	**25** (d)	**26** (c)	**27** (a)	**28** (a)	**29** (a)	**30** (b)
31 (a)	**32** (a)	**33** (a)	**34** (c)	**35** (c)	**36** (a)	**37** (a)	**38** (a)	**39** (c)	**40** (c)
41 (b)	**42** (d)	**43** (b)	**44** (b)	**45** (a)	**46** (a)	**47** (b)	**48** (a)	**49** (b)	**50** (c)

Hints and Explanations

1 $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x,$

Hence, domain of $f(x)$ is ± 1. So, the range is $\{f(1), f(-1)\}$ i.e. $\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$.

2 In ΔPMC, $\tan\alpha = \frac{h-a}{PM}$



$\Rightarrow \quad PM = (h-a)\cot\alpha \quad$ ...(i)

and in $\Delta PMC'$, $\tan\beta = \frac{h+a}{PM}$

$\Rightarrow \; h + a = PM \tan\beta$

$\therefore \; h = (h-a)\cot\alpha \tan\beta - a$ [from Eq. (i)]

$\Rightarrow h(1 - \cot\alpha \tan\beta) = -a(\cot\alpha \tan\beta + 1)$

$\Rightarrow \quad h = \frac{a(\sin\alpha\cos\beta + \cos\alpha\sin\beta)}{\sin\beta\cos\alpha - \sin\alpha\cos\beta}$

$\Rightarrow \quad h = \frac{a\sin(\alpha+\beta)}{\sin(\beta-\alpha)}$ m

3 Since, $0 \le \cos^{-1} x_i \le \pi$

$\therefore \quad \cos^{-1} x_i = 0 \; \forall \, i$

$\therefore \quad x_i = 1, \; \forall \, i$

$\therefore \quad \sum_{i=1}^{2n} x_i = 2n$

4 Now, $\cos^{-1}(\cos 4) = \cos^{-1}[\cos(2\pi - 4)]$

$= 2\pi - 4$

$\therefore \quad \cos^{-1}(\cos 4) > 3x^2 - 4x$

$\therefore \quad 2\pi - 4 > 3x^2 - 4x$

$\Rightarrow \quad 3x^2 - 4x - (2\pi - 4) < 0$

$\Rightarrow \quad \frac{2 - \sqrt{6\pi - 8}}{3} < x < \frac{2 + \sqrt{6\pi - 8}}{3}$

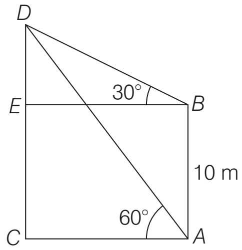
5 Since, $\sin^{-1} x + \cos^{-1} x + \tan^{-1} x = \frac{\pi}{2} + \tan^{-1} x$

Now, $-\frac{\pi}{2} \le \tan^{-1} x \le \frac{\pi}{2}$

$\Rightarrow \quad 0 \le \frac{\pi}{2} + \tan^{-1} x \le \pi$

$\therefore \quad \alpha = 0, \beta = \pi$

6 Let AB and CD be the pole and tower, respectively.



In ΔACD, $\tan 60° = \frac{CD}{AC}$

$\Rightarrow \quad AC = \frac{CD}{\sqrt{3}} \quad$ ...(i)

In ΔDBE, $\tan 30° = \frac{DE}{BE} = \frac{DE}{CA}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{DE}{AC}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{DE}{CD/\sqrt{3}}$ [from Eq. (i)]

$\Rightarrow \quad \frac{CD}{DE} = 3 \quad \Rightarrow \quad \frac{DE + EC}{DE} = 3$

$\Rightarrow \quad DE = \frac{EC}{2} = \frac{10}{2} = 5$ m

$\therefore \quad CD = DE + EC = 10 + 5 = 15$ m

7 $12\sin\theta - 9\sin^2\theta$

$= -(3\sin\theta - 2)^2 + 4 \le 4$

Hence, maximum value is 4.

8 Since, $\sin\alpha + \sin\beta + \sin\gamma$

$= 4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

Also, since each of $\frac{\alpha}{2}, \frac{\beta}{2}$ and $\frac{\gamma}{2}$ is less than $\frac{\pi}{2}$. So, $\cos\frac{\alpha}{2}$, $\cos\frac{\beta}{2}$ and $\cos\frac{\gamma}{2}$ are all positive.

Hence, minimum value of $4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$ is positive.

9 $\therefore \; \cos^{-1}\sqrt{x} + \cos^{-1}\sqrt{1-x}$

$= \cos^{-1}[\sqrt{x}\sqrt{1-x} - \sqrt{1-x}\sqrt{x}]$

$= \cos^{-1}(0) = \frac{\pi}{2}$

$\therefore \quad \pi = \frac{\pi}{2} + \cos^{-1}\sqrt{1-y}$

$\Rightarrow \; \frac{\pi}{2} = \cos^{-1}\sqrt{1-y} \Rightarrow \sqrt{1-y} = 0$

$\therefore \quad y = 1$

10 Since, α and β are the roots of the equation $\quad 5\cos\theta + 4\sin\theta = 3.$

$\therefore \quad 5\cos\alpha + 4\sin\alpha = 3 \quad$ ...(i)

and $\quad 5\cos\beta + 4\sin\beta = 3 \quad$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$5(\cos\alpha - \cos\beta) + 4(\sin\alpha - \sin\beta) = 0$

$\Rightarrow \quad -5 \times 2 \cdot \sin\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2}$

$+ 4 \cdot 2 \cdot \cos\frac{\alpha+\beta}{2} \cdot \sin\frac{\alpha-\beta}{2} = 0$

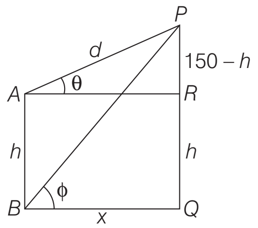
$\Rightarrow \quad \left(4\cos\frac{\alpha+\beta}{2} - 5\sin\frac{\alpha+\beta}{2}\right) = 0$

⇒ $\tan\frac{\alpha+\beta}{2}=\frac{4}{5}$

∴ $\cos(\alpha+\beta)=\dfrac{1-\tan^2\left(\frac{\alpha+\beta}{2}\right)}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)}$

$=\dfrac{1-\frac{16}{25}}{1+\frac{16}{25}}=\dfrac{\frac{9}{25}}{\frac{41}{25}}=\dfrac{9}{41}$

11 Let $AR = x$ and the height of the chimney, $AB = h$.



Now, $PR = PQ - RQ = 150 - h$

In ΔPAR, $\tan\theta=\frac{PR}{AR}$

⇒ $\frac{4}{3}=\frac{150-h}{x}$ ...(i)

and in ΔPBQ, $\tan\phi=\frac{PQ}{BQ}$

⇒ $\frac{5}{2}=\frac{150}{x}$ ...(ii)

∴ $\frac{150-h}{150}=\frac{4}{3}\times\frac{2}{5}=\frac{8}{15}$

[dividing Eq. (i) by Eq. (ii)]

⇒ $1-\frac{h}{150}=\frac{8}{15}\Rightarrow h=70\text{ m}$

From Eq. (i), $\frac{150-70}{x}=\frac{4}{3}$

⇒ $x = 60$ m and

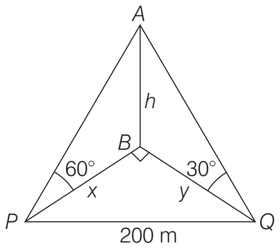
$PR = 150 - 70 = 80$ m

In ΔPAR, $AP^2 = AR^2 + PR^2$

⇒ $d^2 = 60^2 + 80^2$

∴ $d = 100$ m

12 Let the height of the hill be h and let A be its top.



Since, BQ and BP represents South and West, respectively.

In ΔAPB, $\tan 60^\circ=\frac{AB}{PB}$

⇒ $\sqrt{3}=\frac{h}{x}\Rightarrow x=\frac{h}{\sqrt{3}}$

Again, in ΔAQB, $\tan 30^\circ=\frac{AB}{BQ}$

⇒ $\frac{1}{\sqrt{3}}=\frac{h}{y}\Rightarrow y=h\sqrt{3}$

In right angled ΔPBQ,

$PQ^2 = PB^2 + BQ^2 = x^2 + y^2$

⇒ $(200)^2=\frac{h^2}{3}+3h^2=h^2\left(\frac{10}{3}\right)$

⇒ $h^2=200^2\times\frac{3}{10}$

∴ $AB=200\sqrt{\frac{3}{10}}=109.54$ m

13 Now, $\frac{1}{\sqrt{2}}\left(\cos\frac{7\pi}{5}-\sin\frac{2\pi}{5}\right)$

$=\cos\frac{\pi}{4}\cos\frac{7\pi}{5}-\sin\frac{\pi}{4}\sin\frac{2\pi}{5}$

$=\cos\frac{\pi}{4}\cos\frac{7\pi}{5}+\sin\frac{\pi}{4}\sin\frac{7\pi}{5}$

$=\cos\left(\frac{7\pi}{5}-\frac{\pi}{4}\right)=\cos\left(\frac{23\pi}{20}\right)$

$=\cos\left(2\pi-\frac{17\pi}{20}\right)=\cos\left(\frac{17\pi}{20}\right)$

∴ $\cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(\cos\frac{7\pi}{5}-\sin\frac{2\pi}{5}\right)\right\}=\frac{17\pi}{20}$

14 Since, $\sec^2\theta\ge 1\Rightarrow\frac{4ab}{(a+b)^2}\ge 1$ and

$a, b\ne 0$

⇒ $a, b\ne 0$ and $-\frac{(a-b)^2}{(a+b)^2}\ge 0$

⇒ $a, b\ne 0$ and $-(a-b)^2\ge 0$

⇒ $a = b$ and $a\ne 0$

15 Given, $x=\cos^{-1}(\cos 4)$

⇒ $x=\cos^{-1}\cos(2\pi-4)$

⇒ $x=2\pi-4$ and $y=\sin^{-1}(\sin 3)$

⇒ $y=\sin^{-1}\sin(\pi-3)\Rightarrow y=\pi-3$

∴ $x+y=3\pi-7$

∴ $\tan(x+y)=-\tan 7$

16 Since, $\log_2 x\ge 0\Rightarrow x\ge 1$

For $x\ge 1$, we have

$\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\pi-2\tan^{-1}x$

∴ $\log_{1/\pi}\left\{\sin^{-1}\frac{2x}{1+x^2}+2\tan^{-1}x\right\}$

$=\log_{1/\pi}\{\pi-2\tan^{-1}x+2\tan^{-1}x\}$

$=\log_{1/\pi}\pi=-1$

17 ∵ $3\tan A-4=0\Rightarrow\tan A=4/3$

⇒ $\cos A=-\frac{3}{5}$ and $\sin A=-\frac{4}{5}$

[since, A lies in III quadrant]

∴ $\sin 2A=\frac{2\tan A}{1+\tan^2 A}=\frac{24}{25}$

∴ $5\sin 2A+3\sin A+4\cos A$

$=\frac{24}{5}-\frac{12}{5}-\frac{12}{5}=0$

18 $27^{\cos x}+81^{\sin x}=3^{3\cos x}+3^{4\sin x}$

$\ge 2\cdot\sqrt{3^{3\cos x}\cdot 3^{4\sin x}}$ [∵ AM ≥ GM]

$=2\cdot 3^{(3\cos x+4\sin x)/2}\ge 2\cdot 3^{\frac{1}{2}(-5)}$

[∵ $-5\le 3\cos x+4\sin x\le 5$]

$=2\cdot 3^{-\frac{5}{2}}=2\cdot 3^{-2}\cdot 3^{-\frac{1}{2}}$

$=\frac{2}{9\sqrt{3}}$

19 Let $\sin x = t$

∴ $t^2+at+1=0\Rightarrow t+\frac{1}{t}=-a$

⇒ $|a|=\left|t+\frac{1}{t}\right|\ge 2$

⇒ $|a|\ge 2$ [∵ AM ≥ GM]

Hence, for no real solution $|a| < 2$.

20 Given, $\frac{1}{n}=\frac{\sin(\theta+2\alpha)}{\sin\theta}$

On applying componendo and dividendo, we get

⇒ $\frac{1+n}{1-n}=\frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta}$

⇒ $\frac{1+n}{1-n}=\frac{2\sin(\theta+\alpha)\cos\alpha}{2\cos(\theta+\alpha)\sin\alpha}$

$=\tan(\theta+\alpha)\cot\alpha$

⇒ $\frac{1+n}{1-n}\tan\alpha=\tan(\theta+\alpha)$

21 $(\sin^2 x)^3+(\cos^2 x)^3$

$=(\sin^2 x+\cos^2 x)^3-3\sin^2 x\cos^2 x$

$(\sin^2 x+\cos^2 x)$

$=1-3\sin^2 x\cos^2 x=1-\frac{3}{4}(\sin 2x)^2$

∴ Maximum value $=1-\frac{3}{4}\times 0=1$

and minimum value $=1-\frac{3}{4}\times 1=\frac{1}{4}$

22 Now, $\tan\alpha\cdot\tan(2n-1)\alpha$

$=\tan\alpha\tan\left(\frac{\pi}{2\alpha}-1\right)\alpha=\tan\alpha\cot\alpha=1$

Hence, the value of given expression is 1.

23 $2-\cos x+\sin^2 x=2-\cos x$

$+1-\cos^2 x$

$=3-(\cos^2 x+\cos x)$

$=3-\left[\left(\cos x+\frac{1}{2}\right)^2-\frac{1}{4}\right]$

Hence, the maximum value occurs at

$\cos x=-\frac{1}{2}$ and its value

$$= 2 - \left(-\frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) = \frac{13}{4}$$

and minimum value occurs at $\cos x = 1$ and its value $= 2 - 1 + (1 - 1) = 1$.

$\therefore$ Required ratio $= \frac{13}{4}$

24 Given, $\frac{m}{a} = \cos^3\theta + 3\cos\theta\sin^2\theta$

and $\frac{n}{a} = \sin^3\theta + 3\cos^2\theta\sin\theta$

$$\therefore \left(\frac{m}{a} + \frac{n}{a}\right) = (\sin\theta + \cos\theta)^3$$

$$\Rightarrow \left(\frac{m+n}{a}\right)^{1/3} = \sin\theta + \cos\theta$$

and $\left(\frac{m-n}{a}\right)^{1/3} = \cos\theta - \sin\theta$

$$\therefore \left(\frac{m+n}{a}\right)^{2/3} + \left(\frac{m-n}{a}\right)^{2/3} = 2(\sin^2\theta + \cos^2\theta)$$

$$(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$$

25 Given, $a\sin^2\alpha - \frac{1}{a}\operatorname{cosec}^2\alpha = 0$

$$\Rightarrow \sin^2\alpha = \frac{1}{a}$$

$\therefore \cos^2\alpha + 5\sin\alpha\cos\alpha + 6\sin^2\alpha$

$$= 1 - \frac{1}{a} + \frac{5}{\sqrt{a}}\sqrt{1 - \frac{1}{a}} + \frac{6}{a}$$

$$= 1 + \frac{5\sqrt{a-1} - 1}{a} + \frac{6}{a}$$

$$= \frac{a + 5\sqrt{a-1} + 5}{a}$$

26 $\tan\frac{x}{2} = \dfrac{1 + \tan^2\frac{x}{2}}{2\tan\frac{x}{2}} - \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$\Rightarrow 2\tan^2\frac{x}{2}\left(1 + \tan^2\frac{x}{2}\right) = \left(1 + \tan^2\frac{x}{2}\right)^2 - 4\tan^2\frac{x}{2}$$

$\Rightarrow 2y(1+y) = (1+y)^2 - 4y$ $\left[\text{put } \tan^2\frac{x}{2} = y\right]$

$$\Rightarrow y^2 + 4y - 1 = 0$$

$$\therefore y = \frac{-4 \pm \sqrt{16+4}}{2} = -2 \pm \sqrt{5}$$

Since, $y \ge 0$, we get

$$y = \sqrt{5} - 2 = \frac{(\sqrt{5}-2)^2}{\sqrt{5}-2} \cdot \frac{2+\sqrt{5}}{2+\sqrt{5}}$$

$$= (9 - 4\sqrt{5})(2 + \sqrt{5})$$

27 Now, $\sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots + \sqrt{2(1+\cos\theta)}}}}$

$$= \sqrt{2 + \sqrt{2 + \ldots + \sqrt{2 + 2\cos\frac{\theta}{2}}}}$$

....

....

$$= \sqrt{2 + 2\cos\left(\frac{\theta}{2^{n-1}}\right)}$$

$$= \sqrt{2\left(1 + \cos\frac{\theta}{2^{n-1}}\right)}$$

$$= \sqrt{2 \cdot 2\cos^2\left(\frac{\theta}{2 \cdot 2^{n-1}}\right)} = 2\cos\left(\frac{\theta}{2^n}\right)$$

28 In ΔABC, $A + B + C = \pi$

$\therefore \tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\Rightarrow 6 = 2\tan A \Rightarrow \tan A = 3$

$\therefore \tan B + \tan C = 3$

and $\tan B \tan C = 2$

$\Rightarrow \tan B = 1$ or 2 and $\tan C = 2$ or 1

Now, $\sin^2 A = \dfrac{\tan^2 A}{1 + \tan^2 A} = \dfrac{9}{10}$

$$\sin^2 B = \frac{\tan^2 B}{1 + \tan^2 B} = \frac{1}{1+1}, \frac{4}{1+4}$$

$$= \frac{1}{2}, \frac{4}{5} = \frac{5}{10}, \frac{8}{10}$$

and $\sin^2 C = \dfrac{\tan^2 C}{1 + \tan^2 C} = \dfrac{8}{10}, \dfrac{5}{10}$

$\therefore \sin^2 A : \sin^2 B : \sin^2 C$

$$= \frac{9}{10} : \frac{5}{10} : \frac{8}{10} \text{ or } \frac{9}{10} : \frac{8}{10} : \frac{5}{10}$$

29 Maximum value of $\sin\theta$ and $\cos\theta$ is 1.

$\because \cos x \cdot \sin y = 1$

$\Rightarrow \cos x = 1, \sin y = 1$

or $\cos x = -1, \sin y = -1$

$\Rightarrow x = 0, 2\pi, y = \frac{\pi}{2}, \frac{5\pi}{2}$

or $x = \pi, 3\pi, y = \frac{3\pi}{2}$

$\therefore$ Required number of ordered pair $= 2 \times 2 + 2 \times 1 = 6$

30 Given, $(1 - \cos^2 6x) + \sin^4 2x = 0$

$\Rightarrow \sin^2 6x + \sin^4 2x = 0$

$\Rightarrow \sin^2 6x = 0$ and $\sin^4 2x = 0$

$\Rightarrow 6x = n\pi$ and $2x = n\pi$

$\Rightarrow x = \frac{n\pi}{6}$

and $x = \frac{n\pi}{2}$

31 Since, $x^2 + x^{-2} \ge 2$ $[\because \text{AM} \ge \text{GM}]$

therefore the equation is valid only if

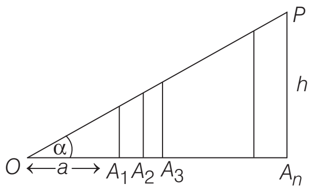
$$2\cos^2\frac{x}{2}\sin^2 x = 2$$

$\Leftrightarrow \cos\frac{x}{2} = \operatorname{cosec} x$

i.e. iff $\operatorname{cosec} x = \cos\frac{x}{2} = 1$

which cannot be true.

32 Consider $A_1, A_2, \ldots, A_n$ be the foot of the n poles subtending angle α to O, such that $OA_1 = a$, if d be the distance between two consecutive poles



$OA_2 = a + d$

$OA_3 = a + 2d$

$\vdots \quad \vdots \quad \vdots$

$OA_n = a + (n-1)d$

Now, in ΔPOA_n,

$$\tan\alpha = \frac{h}{OA_n}$$

$$OA_n = h\cot\alpha$$

$$\Rightarrow a + (n-1)d = h\cot\alpha$$

$$d = \frac{h\cot\alpha - a}{n-1}$$

$$= \frac{h\cos\alpha - a\sin\alpha}{(n-1)\sin\alpha}$$

33 $\tan^{-1}\left(\dfrac{(x+1)+(x-1)}{1-(x+1)(x-1)}\right) = \tan^{-1}\left(\dfrac{8}{31}\right)$

Provided $(x+1)(x-1) < 0$

i.e. $x^2 < 1$ ...(i)

$$\Rightarrow \tan^{-1}\frac{2x}{1-(x^2-1)} = \tan^{-1}\left(\frac{8}{31}\right)$$

$$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31}$$

$$\Rightarrow 4x^2 + 31x - 8 = 0$$

$$\Rightarrow (4x-1)(x+8) = 0$$

$$\Rightarrow x = \frac{1}{4} \text{ or } -8 \Rightarrow x = \frac{1}{4}$$

Since, $x = -8$ is not satisfied the Eq. (i).

34 Obviously $x > 0$ and $x\sqrt{3} < 1$

i.e. $x < \frac{1}{\sqrt{3}}$

If $x > \frac{1}{\sqrt{3}}$, then $\cos^{-1}(x\sqrt{3})$ will be undefined. If $x < 0$, then $x\sqrt{3} < 0$. Hence,

$\cos^{-1} x > \frac{\pi}{2}$ and $\cos^{-1} x\sqrt{3} > \frac{\pi}{2}$

which is not satisfied the equation.

$$\therefore x \in \left(0, \frac{1}{\sqrt{3}}\right)$$

Given, $\cos^{-1}(x\sqrt{3})$

$$= \frac{\pi}{2} - \cos^{-1} x = \sin^{-1} x$$

$$\Rightarrow \cos^{-1}(x\sqrt{3}) = \cos^{-1}\sqrt{1-x^2}$$

$$\Rightarrow x\sqrt{3} = \sqrt{1-x^2}$$

$$\Rightarrow 3x^2 = 1 - x^2 \Rightarrow x = \pm\frac{1}{2}$$

$x = \frac{1}{2}$ $\left[\because x = -\frac{1}{2} \notin \left(0, \frac{1}{\sqrt{3}}\right)\right]$

35 $\because \dfrac{1+\sin x}{\cos x} = 2\cos x, \cos x \neq 0$

$\therefore \quad 1 + \sin x = 2(1 - \sin^2 x)$

$\Rightarrow \quad 1 + \sin x = 2(1 + \sin x)(1 - \sin x)$

$\Rightarrow \quad (1 + \sin x)[1 - 2(1 - \sin x)] = 0$

$\Rightarrow \quad \sin x = \dfrac{1}{2}, -1$

$\therefore \quad x = \dfrac{\pi}{6}, \pi - \dfrac{\pi}{6}, \dfrac{3\pi}{2}$

$x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, x \neq \dfrac{3\pi}{2}$ $[\because \cos x \neq 0]$

36 Given, $\cot^2 x + \operatorname{cosec} x - a = 0$

$\Rightarrow \operatorname{cosec}^2 x + \operatorname{cosec} x - 1 - a = 0$

$\Rightarrow \left(\operatorname{cosec} x + \dfrac{1}{2}\right)^2 = 1 + a + \dfrac{1}{4} = \dfrac{5}{4} + a$

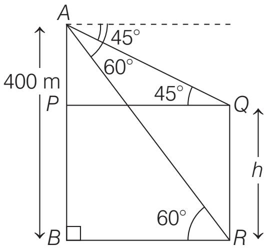
$\therefore \quad \operatorname{cosec} x \geq 1$ or ≤ -1

$\Rightarrow \quad \operatorname{cosec} x + \dfrac{1}{2} \geq \dfrac{3}{2}$ or $\leq -\dfrac{1}{2}$

$\Rightarrow \left(\operatorname{cosec} x + \dfrac{1}{2}\right)^2 \geq \dfrac{1}{4} \Rightarrow \dfrac{5}{4} + a \geq \dfrac{1}{4}$

$\therefore \quad a \geq -1$

37 Let h be the height of the tower QR.



Then, $PA = 400 - h$

In ΔAPQ, $\dfrac{AP}{PQ} = 1 \Rightarrow AP = PQ$

$\Rightarrow \quad 400 - h = PQ$

Again in ΔABR, $\tan 60° = \dfrac{400}{BR}$

$[\because BR = PQ]$

$\therefore \quad \sqrt{3} = \dfrac{400}{400 - h}$

$\Rightarrow \quad 400\sqrt{3} - h\sqrt{3} = 400$

$\Rightarrow \quad (400\sqrt{3} - 400) = h\sqrt{3}$

$\Rightarrow \quad \dfrac{400(\sqrt{3}-1)}{\sqrt{3}} = h$

$= \dfrac{400(3 - \sqrt{3})}{3}$ m

$= 169.06$ m

38 $\cos 5\theta = \cos(3\theta + 2\theta)$

$= \cos 3\theta \cdot \cos 2\theta - \sin 3\theta \cdot \sin 2\theta$

$= (4\cos^3\theta - 3\cos\theta)(2\cos^2\theta - 1)$

$-(3\sin\theta - 4\sin^3\theta) \times (2\sin\theta \cdot \cos\theta)$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta$

$- 2\sin^2\theta(3 - 4\sin^2\theta) \cdot \cos\theta$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta$

$- 2\cos\theta(4\cos^2\theta - 1)(1 - \cos^2\theta)$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta$

$- 2\cos\theta[4\cos^2\theta - 4\cos^4\theta$

$- 1 + \cos^2\theta]$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

On comparing the coefficient of $\cos^5\theta$, $\cos\theta$ and constant term, we get $P = 5, Q = 16$ and $R = 0$

$\therefore \quad P + Q + R = 21$

39 $\because \dfrac{\cos A}{a} = \dfrac{\cos B}{b} = \dfrac{\cos C}{c}$

$\therefore \quad \dfrac{\cos A}{k\sin A} = \dfrac{\cos B}{k\sin B} = \dfrac{\cos C}{k\sin C}$

$\Rightarrow \quad \cot A = \cot B = \cot C$

$\Rightarrow \quad A = B = C = 60°$

40 $a^2 \sin 2B + b^2 \sin 2A$

$= 2a^2 \sin B \cdot \cos B + 2b^2 \sin A \cdot \cos A$

$= \dfrac{a^2 b}{R}\cos B + \dfrac{b^2 a}{R}\cos A$

$= \dfrac{ab}{R}(a\cos B + b\cos A) = \dfrac{abc}{R}$

$= 2bc\sin A = 4\left(\dfrac{1}{2}bc\sin A\right)$

$= 4\lambda$

41 Given, $\cos 2x + 2\cos x = 1$

$\Rightarrow \quad 2\cos^2 x - 1 + 2\cos x - 1 = 0$

$\Rightarrow \quad \cos^2 x + \cos x - 1 = 0$

$\Rightarrow \quad \cos x = \dfrac{-1+\sqrt{5}}{2}$

$\left[\text{neglecting } \dfrac{-1-\sqrt{5}}{2}, \text{As } -1 \leq \cos x \leq 1 \right.$

$\left.\text{and} \left(\dfrac{-1-\sqrt{5}}{2}\right) < -1\right]$

$\therefore \quad \cos^2 x = \left(\dfrac{\sqrt{5}-1}{2}\right)^2$

$= \dfrac{6 - 2\sqrt{5}}{4} = \dfrac{3 - \sqrt{5}}{2}$

$\therefore \quad \sin^2 x(2 - \cos^2 x)$

$= \left(1 - \dfrac{3-\sqrt{5}}{2}\right)\left(2 - \dfrac{3-\sqrt{5}}{2}\right)$

$= \left(\dfrac{\sqrt{5}-1}{2}\right)\left(\dfrac{\sqrt{5}+1}{2}\right) = 1$

42 From the given parts of question, we get

$\cos x + \sin x = A - 1 = B + 1$

$\Rightarrow \quad A = B + 2$ ... (i)

and $\quad A \cdot B = (\sin x + \cos x + 1)$

$(\sin x + \cos x - 1)$

$= (\sin x + \cos x)^2 - 1$

$1 + \sin 2x - 1 = \sin 2x$

$\Rightarrow \quad (B + 2) \cdot B = \sin 2x$ [from Eq. (i)]

$\Rightarrow \quad B^2 + 2B - \sin 2x = 0$

$\Rightarrow \quad (A - 2)^2 + 2(A - 2) - \sin 2x = 0$

$\Rightarrow \quad A^2 - 2A - \sin 2x = 0$

43 Given equation is

$x^3 - 15x^2 + 47x - 82 = 0$

$\sum a = 15$

$\sum ab = 47$

$abc = 82$

Now, $\dfrac{\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{\cos C}{c}$

$\dfrac{b^2 + c^2 - a^2}{2abc} + \dfrac{c^2 + a^2 - b^2}{2abc} + \dfrac{a^2 + b^2 - c^2}{2abc}$

(by cosine rule)

$= \dfrac{a^2 + b^2 + c^2}{2abc}$

$= \dfrac{(\sum a)^2 - 2\sum ab}{2abc}$

$= \dfrac{225 - 94}{2 \cdot 82} = \dfrac{131}{164}$

44 Given, $a^4 + b^4 + c^4 - 2b^2c^2 - 2c^2a^2 = 0$

$\Rightarrow \quad (a^2 + b^2 - c^2)^2 = 2a^2b^2$

$\Rightarrow \quad \dfrac{(a^2 + b^2 - c^2)^2}{4a^2b^2} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{a^2 + b^2 - c^2}{2ab} = \pm\dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad \cos C = \pm\dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad C = \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$

So, the angle is 45° or 135°.

45 $f(\theta, \alpha) = 2\sin^2\theta + 4\cos(\theta + \alpha)$

$\sin\theta\sin\alpha + 2\cos^2(\theta + \alpha) - 1$

$= 2\sin^2\theta + 2\cos(\theta + \alpha)$

$[2\sin\theta\sin\alpha + \cos(\theta + \alpha)] - 1$

$= 2\sin^2\theta + 2\cos(\theta + \alpha)$

$[\sin\theta\sin\alpha + \cos\theta\cos\alpha] - 1$

$= 2\sin^2\theta + 2\cos(\theta + \alpha)\cos(\theta - \alpha) - 1$

$= 2\sin^2\theta + 2\cos^2\theta - 2\sin^2\alpha - 1$

$= 1 - 2\sin^2\alpha = \cos 2\alpha$

$\therefore \quad f\left(\dfrac{\pi}{3}, \dfrac{\pi}{4}\right) = \cos\left(2 \times \dfrac{\pi}{4}\right) = 0$

46 I. The general value of θ satisfying any of the equations

$\sin^2\theta = \sin^2\alpha, \cos^2\theta = \cos^2\alpha$

and $\tan^2\theta = \tan^2\alpha$ is given by

$\theta = n\pi \pm \alpha$.

II. The general value of θ satisfying equations $\sin\theta = \sin\alpha$ and $\cos\theta = \cos\alpha$ simultaneously is given by $\theta = 2n\pi + \alpha$, $n \in Z$.

So, Statement I is correct and Statement II is incorrect.

47 **Statement I** Put $x = \cos\theta$, then $0 \le \theta \le \frac{\pi}{3}$

$$\text{LHS} = \cos^{-1}(\cos\theta) - \sin^{-1}\left[\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right]$$

$$= \theta - \sin^{-1}\left[\sin\left(\theta + \frac{\pi}{3}\right)\right]$$

$$= \theta - \theta - \frac{\pi}{3} = -\frac{\pi}{3}$$

Statement II Put $x = \sin\theta$,

then $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

$$\text{LHS} = \sin^{-1}(2\sin\theta\cos\theta)$$
$$= \sin^{-1}(\sin 2\theta)$$
$$= 2\theta = 2\sin^{-1}x$$

48 Given, $2\sin^2\theta - \cos 2\theta = 0$

$$\Rightarrow 4\sin^2\theta = 1 \Rightarrow \sin\theta = \pm\left(\frac{1}{2}\right)$$

So, $\sin\theta = \frac{1}{2}$

[$\because \sin\theta = -\frac{1}{2}$ does not satisfy the second equation]

$$\therefore \quad \theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

which also satisfy $2\cos^2\theta - 3\sin\theta = 0$.

Hence, the number of solutions are two.

49 Given, $2\sin\left(\frac{\theta}{2}\right) = \sqrt{\left[\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right]^2} + \sqrt{\left[\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right]^2}$

$$= \left|\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right| + \left|\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right|$$

$$\Rightarrow \quad \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right) > 0$$

$$\Rightarrow \quad \sin\left(\frac{\theta}{2} + \frac{\pi}{4}\right) > 0$$

and $\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right) < 0$

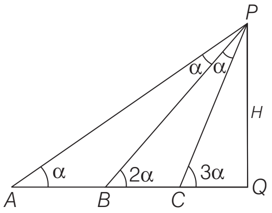
and $\cos\left(\frac{\theta}{2} + \frac{\pi}{4}\right) < 0$

$$\Rightarrow \quad 2n\pi + \frac{\pi}{2} < \frac{\theta}{2} + \frac{\pi}{4} < 2n\pi + \pi$$

$$\therefore \quad 2n\pi + \frac{\pi}{4} < \frac{\theta}{2} < 2n\pi + \frac{3\pi}{4}$$

50 **Statement I** $\angle APB = 2\alpha - \alpha = \alpha$

and $\angle BPC = 3\alpha - 2\alpha = \alpha$



Hence, PB is an angle bisector of $\angle APC$.

Then, $\frac{AB}{BC} = \frac{AP}{CP}$

$$= \frac{H \operatorname{cosec}\alpha}{H \operatorname{cosec} 3\alpha}$$

$$= \frac{\sin 3\alpha}{\sin\alpha}$$

Statement II But Statement II is not always true.